Resonance Model for $\gamma+N\rightarrow Y+K$†

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A model which includes a resonance proposed by Kanazawa is used to describe the low-energy behavior of the reaction $\gamma+N\rightarrow K+\Lambda^0$. Only those contributions from the N, K, K^*, and $p_{1/2}$ nucleon resonance of Kanazawa are considered. The resonance is centered at 1718 MeV with a full width of 120 MeV. The parameters in the model are the coupling constants at the $N-\Lambda^0-K$ and $N-\Lambda^0-K^*$ vertices, the form factors at the $\gamma-K-K^*$ vertex, and the height of the resonance. These parameters are evaluated by fitting the Cornell differential cross-section data for the process $\gamma+p\rightarrow\Lambda^0+K^+$. The use of these parameters does not yield results in agreement with the one measured value of polarization of the Λ^0.

INTRODUCTION

THE theory of photoproduction of hyperons has been studied by several authors.[1–7] This process is inherently more difficult to treat at low energies than pion photoproduction since many more channels are open at the threshold of K production as compared to π production. In an effort to achieve some balance between simplicity and realism, we have chosen to consider a model for the process

$$\gamma+p\rightarrow\Lambda^\circ+K^+, \quad (1)$$

which includes the contributions from the nucleon, K, and K^* poles together with a $T=\frac{1}{2}$, $p_{1/2}$ nucleon resonance in the neighborhood of 1700 MeV, which was introduced by Kanazawa[8] in his study of the process

$$\pi^-+p\rightarrow\Lambda^0+K^0.$$

With this model the production amplitude, the differential cross section, and the Λ^0 polarization for unpolarized photons and protons are computed as functions of the barycentric energy and angle of production together with the parameters of the model, which we designate as $g_{N\Lambda K}$, C_1, C_2, F, and s_0, where $g_{N\Lambda K}$ is the coupling constant for pseudoscalar K particles, C_1 and C_2 are two constants arising from the K^* spin 1 pole, and F and s_0 are the height and position, respectively, of the $p_{1/2}$ resonance. We have adjusted our notation to be identical to that of Kuo,[7] who gives explicit definitions of these parameters. We fix the full width of the resonance to be Kanazawa's value of 120 MeV, and we find it convenient to locate the resonance, which Kanazawa placed in the neighborhood of 1700 MeV, at 1718 MeV or in terms of the square of the energy, s_0, at 2.952 BeV.[2]

RESULTS

The barycentric differential cross section for unpolarized photons and protons computed for the model under consideration can be expanded in powers of $\cos\theta$, the direction of the K^+,

$$d\sigma/d\Omega=a_0+a_1\cos\theta+a_2\cos^2\theta+\cdots, \quad (2)$$

where the a_i are functions of $g_{N\Lambda K}$, C_1, C_2, F, and the total barycentric energy W, or equivalently the laboratory photon energy k_L.

To fix the parameters of the model we first plot curves of constant a_i for several energies in the C_1–C_2 plane for $F=0$ in an attempt to see if a fit can be obtained without a resonance. The curves of constant a_0 and a_2 are ellipses while those of constant a_1 are hyperbolas in the C_1–C_2 plane. It is not possible with $F=0$ to obtain reasonable fits to $d\sigma/d\Omega$ at all five energies for which data are available.[9] If $F=0$, reasonable fits for a_0 and a_1 can be obtained only if $a_2<0$ at photon laboratory energies of 1018 and 1054 MeV, whereas the data show that $a_2\gtrsim0$. This result holds no matter what value of $g_{N\Lambda K}$ is used. Thus, the Born terms retained are not sufficient to fit the data. This is to be expected because there is a sizable Λ polarization observed[10] and there will be no polarization of the outgoing Λ particle if only the pole terms are kept.

Although a fit cannot be obtained for $F=0$, we use the curves of constant a_i for this case at 976 and 1003

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[7] T. K. Kuo, Phys. Rev. **129**, 2264 (1963). We were about three fourths of the way through our analysis when we received the preprint of Kuo's paper. Our model is identical to his $p_{1/2}$ resonance model. We have independently derived his equations (25)–(30), (34), and (36). However, when we confront these equations as determined by the model with the experimental data (see Ref. 9), we do not find quite the same values of the parameters he lists in his Table II. We, thus, present the results of our investigation without giving any of the underlying formalism since this all appears in Kuo's paper. The thesis of one of us (N.A.B.) contains our considerations of this problem. For other work on the isobar model in associated photoproduction of strange particles see M. Gourdin, Nuovo Cimento **20**, 1035 (1961), and J. Dufour and M. Gourdin, Nuovo Cimento **27**, 1410 (1963).
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TABLE I. Numerical values of a_i in Eq. (2) in 10^{-31} cm²/sr.

k_L(BeV)	a_0	a_1	a_2
0.940	0.93	0.25	0.04
0.976	1.26	0.46	0.08
1.003	1.42	0.64	0.11
1.018	1.51	0.74	0.12
1.054	1.68	0.89	0.09
1.080	1.82	0.91	0.03
1.100	1.95	0.90	−0.05
1.150	2.26	0.84	−0.34

TABLE II. Polarization $P(\cos\theta)$ of the photoproduced Λ.

k_L(BeV)	P
0.940	$\dfrac{0.03\sin\theta(1+0.26\cos\theta+0.01\cos^2\theta)}{1+0.27\cos\theta+0.04\cos^2\theta}$
1.054	$\dfrac{0.21\sin\theta(1+0.55\cos\theta+0.21\cos^2\theta)}{1+0.53\cos\theta+0.05\cos^2\theta}$
1.150	$\dfrac{0.22\sin\theta(1+0.64\cos\theta+0.24\cos^2\theta)}{1+0.37\cos\theta-0.15\cos^2\theta}$

MeV to determine the ranges in which C_1, C_2, and $g_{N\Lambda K}$ are most likely to fall. We assume the effect of the resonance to be small at these energies and find the following ranges to be the best ones for fitting the lowest energy data:

$$4.0<(g^2_{N\Lambda K}/4\pi)<7.0\,, \tag{3a}$$

$$0.75(\text{BeV})^{-1}<C_1<1.50(\text{BeV})^{-1}\,, \tag{3b}$$

$$0.10(\text{BeV})^{-2}<C_2<0.60(\text{BeV})^{-2}\,. \tag{3c}$$

We restrict ourselves to these ranges.

We next determine the sign of F. For $F<0$ and for C_1, C_2, and $g_{N\Lambda K}$ as given above, we cannot obtain reasonable fits to a_0, and a_1 at all energies while keeping a_2 at 1018 and 1054 MeV greater than zero. Therefore, we restrict ourselves to positive F.

Our general procedure with a nonzero resonance is to fit the 1003, 1018, and 1054 MeV data and then check the resulting expressions at 976 and 1080 MeV. The reason for concentrating on the former three energies mentioned above is that there are more data to work with at those energies.

In order to fit the a_i quantities at 1018 and 1054 MeV, we find that $g^2_{N\Lambda K}/4\pi$ must be in the neighborhood of 5.5 to 6.0 and F must be in the neighborhood of $0.010(\text{BeV})^{-1}$. However, if F has this large a value, one obtains values of a_1 at 976 and 1003 MeV that are too high to be in good agreement with the data. Reasonable fits to the cross section data at 1003, 1018, and 1054 MeV can be obtained for the following choice of parameters:

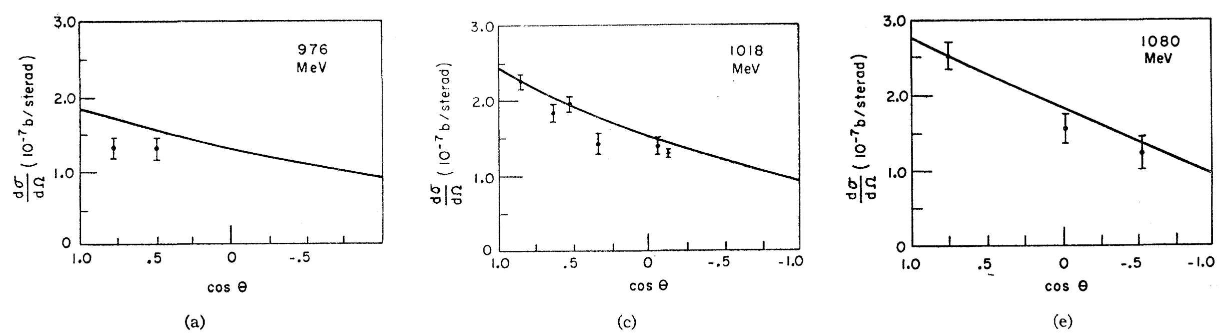
$$g^2_{N\Lambda K}/4\pi=5.8\,, \tag{4a}$$

$$C_1=1.0(\text{BeV})^{-1}\,, \tag{4b}$$

$$C_2=0.40(\text{BeV})^{-2}\,, \tag{4c}$$

$$F=0.006(\text{BeV})^{-1}\,. \tag{4d}$$

We use Eqs. (4) to calculate $d\sigma/d\Omega$ and the polarization P of the produced Λ at several energies with results which are presented in Table I for $d\sigma/d\Omega$ and Table II for P. The polarization is defined with respect



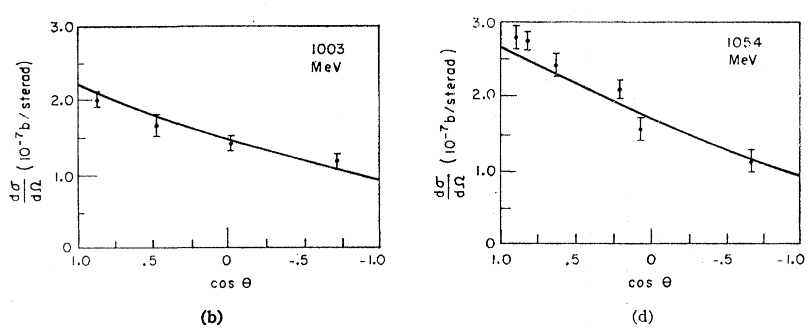


FIG. 1. Differential cross section computed with the model are given by the solid curves for various values of the photon laboratory energy. The points are data from Ref. 9.

to the direction $\mathbf{P}_\gamma \times \mathbf{P}_K$. A graphical comparison of $d\sigma/d\Omega$ with the data is shown in Fig. 1(a)–(e). At $\theta = 80°$, and $k_L = 1.054$ BeV we find the Λ^0 20% polarized in the $\mathbf{P}_\gamma \times \mathbf{P}_K$ direction, which contradicts the measured value of 0.37 ± 0.17 in the $\mathbf{P}_K \times \mathbf{P}_\gamma$ direction[10] ($|\alpha| P = 0.23 \pm 0.08$, and $|\alpha| = 0.62 \pm 0.07$).[11]

SUMMARY AND CONCLUSIONS

We have introduced a model for low-energy photoproduction of Λ particles off protons. We retained only the contributions from the 1-N, 1-K, 1-K^*, and $p_{1/2}$ nucleon resonance intermediate states. The result given by (4a) for $g_{N\Lambda K}{}^2/4\pi = 5.8$ is in acceptable agreement with Kanazawa's[8] value of 5.0. Reasonable agreement with the experimental data for the differential cross sections is obtained at most energies. However, we note that our fits to the differential cross section have one defect: It would appear that a smaller a_1 at the lower energies and slightly higher a_1 at energies above 1020 MeV would improve the fit. The model cannot supply this need while keeping the quantities a_0 and a_2 reasonable.

The model has been used to predict differential cross sections at three energies for which no data are yet available. It has also been used to predict Λ-polarization curves at three different energies, with results that contradict the datum. If this measured value of the polarization is confirmed, then it would be necessary to forsake this model.

It should be remembered that the hyperon poles and resonances and the various nucleon resonances have not been included in this study. In particular, the $f_{5/2}$ nucleon resonance at 1690 MeV, even with its large centrifugal barrier, may be instrumental in producing large Λ polarizations. We are, therefore, continuing our investigations to assess the effects of such contributions on photoproduction and related processes.

We are most grateful to J. E. Rush for checking some of the numerical calculations.

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Gauge Invariance and Integration Rules*

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The Feynman-Dyson rules of integration for the scattering matrix elements in perturbation expansion have long been known to lead to gauge-*variant* results in the case of certain closed loop diagrams. It is shown that a more careful integration which avoids unjustified interchanges of integrations and limiting processes keeps the theory gauge-invariant throughout, without the need of explicit cutoffs or appeal to invariance for the specification of undefined integrals. The Feynman-Dyson rules can thus easily be amended to assure gauge invariance.

THE work of Tomonaga, Schwinger, and Feynman about 15 years ago led to a milestone in the development of quantum field theory: It was finally possible to predict observable effects due to radiative corrections. The success of their work must be attributed, to a large extent, to the extensive use of the invariance properties of the theory. For this reason, it seems contradictory to observe that this same theory is unable to provide gauge-invariant results without explicit help by the "better-knowing" theoretician. The present paper is intended to remedy this situation.

The difficulty appears in those calculations which involve divergent closed loop diagrams. Specifically, the closed loops with two corners lead to a nonvanishing photon self-energy and the one with four corners provides terms to the photon-photon scattering cross section which depends on the potentials rather than the fields. While a gauge-independent quantum electrodynamics, based on field strengths,[1] can be formulated in a covariant manner[2] and would, therefore, avoid this difficulty, there is no reason why the usual Schwinger-Feynman-Dyson formulation should not carry through in a gauge-invariant way. Although the fundamental equations are gauge invariant, the usual integrations result in gauge-dependent terms, so that this invariance property must have been lost in the integration process.

The usual attitude is to consider oneself helpless in view of the appearing divergences, which can easily be blamed for this difficulty as they have been blamed for

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